Title: Tangents, Secants, and Chords…OH MY!

Brief Overview:

Using Geometer's Sketchpad, students will discover and prove proportions involving intersecting chords, secant segments and tangent segments. These lessons are primarily self-guided by the student.

NCTM Content Standard:

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Grade/Level:

High School Geometry

Duration/Length:

One period (45 minutes)

Student Outcomes:

Students will:

- Review properties of similar triangles.
- Review properties relating to measures of inscribed, central, external, and internal angles of circles.
- Discover the proportions relating segments on chords, secants, and tangents of a circle.
- Generate theorems based on observations.
- Prove theorems deductively.

Materials and Resources:

- Geometer's Sketchpad
- Worksheet #1
- Worksheet #2
- Worksheet #3

- Extension
- Enrichment
- Application

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Tangents, Secants, and Chords…OH MY!

Development/Procedures:

Lesson #1: Using Similar Triangles to Identify Chord Relationships

(Overview) In this lesson, students will use Geometer's Sketchpad to demonstrate an application of similar triangles. They will discover the theorem when two chords intersect in a circle, the product of the segments of one chord equals the product of the segments of the other chord. Students may work with a partner or individually.

(Preassessment/Launch 0-5 min) To begin the class, hand out Worksheet #1 to each student. Have students complete the review questions and then discuss the review questions as a class.

(Teacher Facilitated/Student Application 30-40 min). Once the students have reviewed prerequisite concepts, have them continue the worksheet and work self-guided for the rest of the class period. The teacher should offer help if students have difficulty with the Sketchpad constructions, but should encourage them to work through the problems and make conjectures on their own.

(Embedded Assessment) Monitoring student progress on the worksheet is an ongoing assessment.

(Reteaching/Summarizing 0-5 min) After problem #13, bring the class together and ask several students to share their measurements for their chord segments and the resulting products. Ask several students to verbalize their theorems and as a class, decide on a final statement of the theorem. Make sure that the final theorem is mathematically correct and clearly stated for all the students. Then have students complete #14.

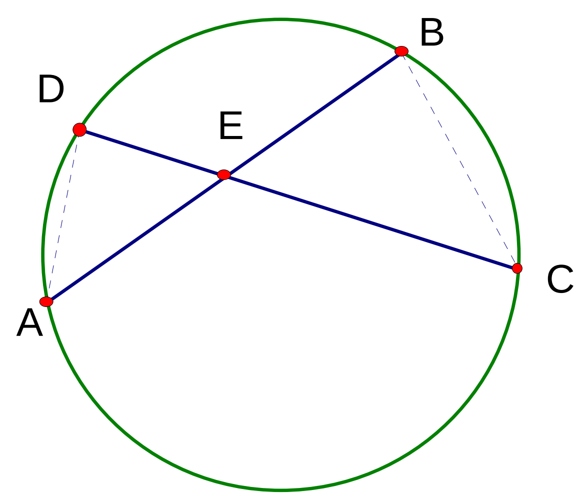
Extension is provided for students who finish early and/or for future class time.

Worksheet #1: Using Similar Triangles to Identify Chord Relationships

Review (Complete the following sentences):

1. **The measure of internal angle <CEA = 1/2 ______________.**
2. **Two inscribed angles that intercept the same arc are __________________.**

3. **If < ADC = 88°, then arc AC = __________.**
4. **If arc DB= 114° and <ADC= 88°, then <DEB = __________.**



1. Open Geometer's Sketchpad.

2. Construct a circle.

3. Construct two intersecting chords that do NOT pass through the center of the circle.

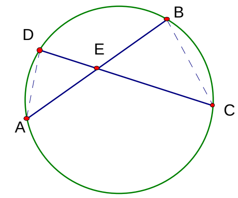
4. Construct the point of intersection of the two chords. Label the endpoints and the point of intersection as shown in the diagram above.

5. Construct the segments that connect the endpoints of each chord as shown in the diagram. By doing this, you create two triangles.

6. **Measure** the following angles and record your answers:
   <ADC = _______ <DAB = _______
   <ABC = _______ <DCB = _______

   Is there a relationship between the angles? How could you have predicted this result without measuring the angles?

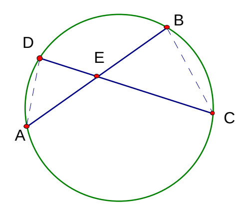
What do you know about <AED and <CEB? Why?



7. Complete the following statement: △DEA ~ △______
   Use mathematics to justify your conclusion.

8. From the definition of similar triangles, corresponding sides are in proportion.

Fill in the following ratios:



$$\frac{CE}{\_\_\_} = \frac{BE}{\_\_\_}$$

9. Rewrite the proportion above as a new equation without fractions.

10. **Measure** the following segments:

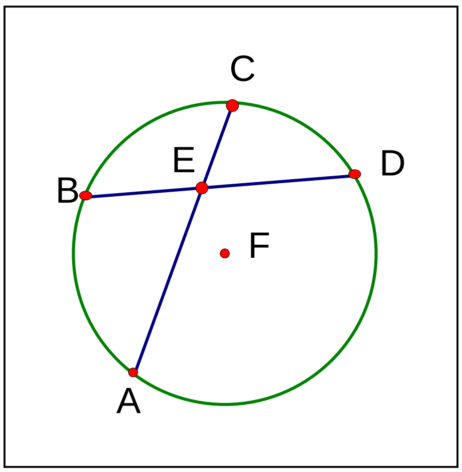
CE = _____ AE = _____
ED = _____ EB = _____

11. Using the Calculate option* in Sketchpad, verify your equation from step 9. Write your numerical results here:

12. Will this relationship always hold true? Drag point A to a new point on the circle and observe your results. Explain.

13. In your own words, write a theorem describing when two chords intersect in a circle.

14. Try the following problems.

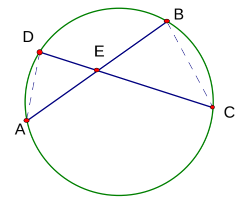


Using the diagram on the Left

1. If BE= 4, CE=3, and DE=9, then AE= __________.

2. If CA= 14, BE= 4, and ED= 6, then AE=________.

*In Sketchpad, Click on Measure and then on Calculate. A box that resembles a calculator will appear on your screen. Since you have already calculated the lengths of the segments, you can tell the calculator to perform functions on selected segments. Click on a segment's length and it will appear in the calculator.



Extensions

1. Ratio of the Areas

a) How do you think the ratio of the areas of the two similar triangles compare to the scale factor?

b) Using the same construction from Worksheet #1, calculate:

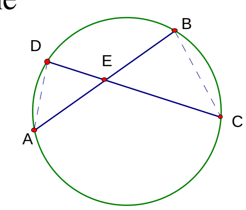
Area of △AED = ________

Area of △CEB = ________

Ratio of Areas = $\dfrac{\text{Area of } \triangle AED}{\text{Area of } \triangle CEB}$ = _______

Scale Factor =

c) Do you notice a relationship between the ratio of the areas of the triangles and the scale factor? If you do not see one, try experimenting with powers and exponents.



2. Maximum and minimum chord products

a) Using your construction, create a table that includes the following segment measurements: AE, EB, DE, EC, AE*EB and DE*EC. Create a total of three different rows by moving the chords around. Observe the change in product values.

AE	EB	DE	EC	AE·EB	DE·EC

b) Continue to move the endpoints of the chords around the circle until you obtain the maximum value for **AE·EB** and **DE·EC**. Add this row to your table above. What do you notice about AE, EB, DE, and EC?

c) Complete the following sentences. When the product of the chords is at the greatest value, the chord segments are also ____________. Point ____ is the center of the circle.

Tangents, Secants, and Chords…OH MY!
Development/Procedures:

Lesson #2: Using Similar Triangles to Identify Secant Relationships

Overview: In this lesson, students will use Geometer's Sketchpad to demonstrate an application of similar triangles. They will discover the theorem when two secant segments are drawn to a circle from an external point, the produce of one secant segment and its external segment equals the product of the other secant segment and its external segment. Students may work with a partner or individually.

(Preassessment/Launch 0-5 min) To begin the class, hand out Worksheet #1 to each student. Have students complete the review questions and then discuss the review questions as a class.

(Teacher Facilitated/Student Application 30-40 min). Once the students have reviewed prerequisite concepts, have them continue the worksheet and work self-guided for the rest of the class period. The teacher should offer help if students have difficulty with the Sketchpad constructions, but should encourage them to work through the problems and make conjectures on their own.

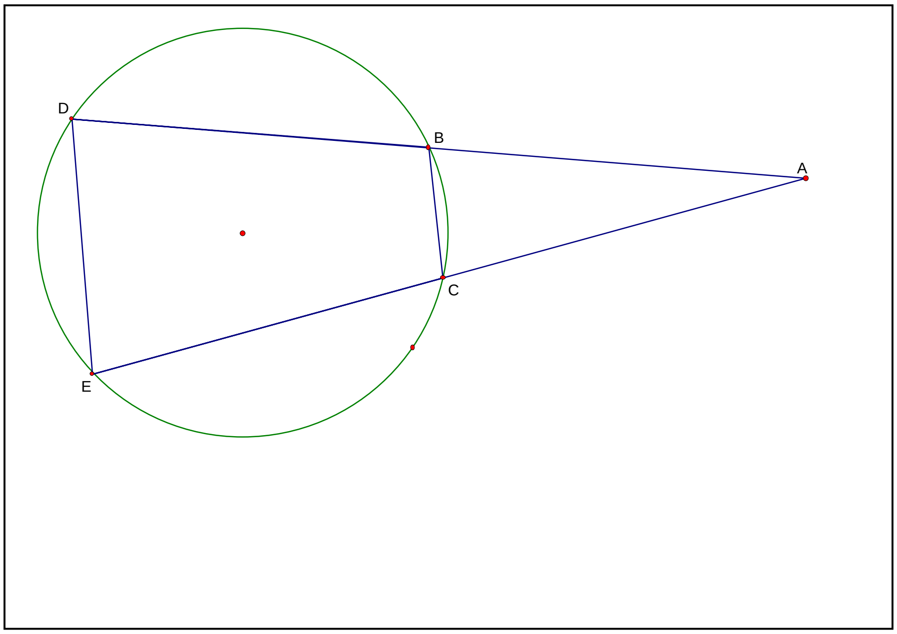
(Embedded Assessment) Monitoring student progress on the worksheet is an ongoing assessment.

(Reteaching/Summarizing 0-5 min) After problem #11, bring the class together and ask several students to share their measurements for their secant segments and the resulting products. Ask several students to verbalize their theorems and as a class, decide on a final statement of the theorem. Make sure that the final theorem is mathematically correct and clearly stated for all the students. Then have students complete #12.

Enrichment is provided for students who finish early and/or for future class time.

Worksheet #2: Using Similar Triangles to Identify Secant Relationships

Review: The measure of inscribed angle <EDB = 1/2 ______________.
The measure of angle <DEC = ½ (________________).
A circle has __________ degrees.
A semicircle has _________ degrees.
The sum of the angles in a triangle is _________ degrees.
A straight angle is ___________ degrees.
Two angles whose measures sum to 180 are _______________.



1. Open Geometer's Sketchpad.

2. Construct the diagram above.

3. **Comlete** the following:

a. <EDB intercepts arc__________. The sum of the two arcs is
b. <ECB intercepts arc _________. ________ degrees.

c. < DEC intercepts arc ________. The sum of the two arcs is
d. <DBC intercepts arc ________. ________ degrees.

4. **Measure** the following angles:

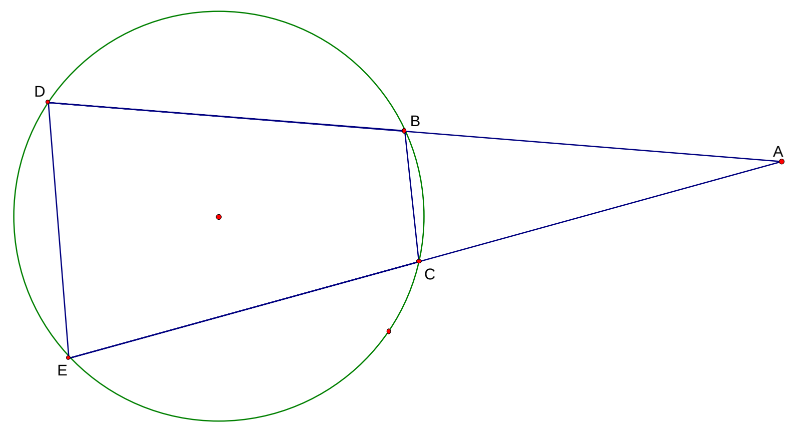
<EDB = _______ <DBC = _______
<ECB = _______ <DEC = _______

Is there a relationship between the angles? How could you have predicted this without measuring the angles?

5. **Measure** the following angles:

<EDA=________ <DEA=__________
<BCA= _______ <CBA=__________

Is there a relationship between the angles? How could you have predicted this without measuring the angles?

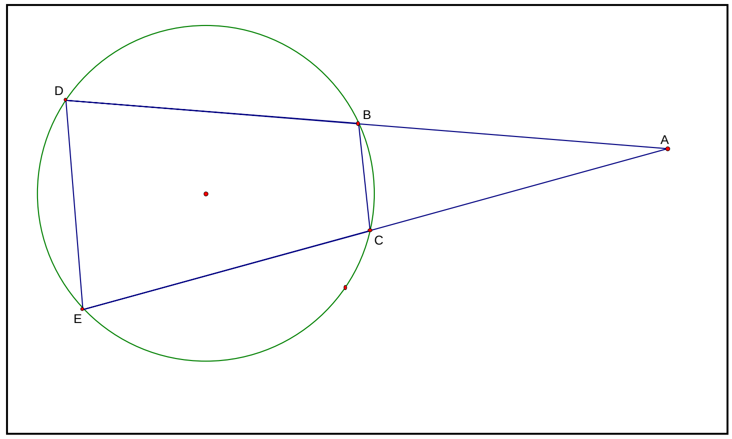


6. Complete the following statement:
△ADE ~ △______.

Use mathematics to justify your statement.

7. From the definition of similar triangles, corresponding sides are in proportion.

Fill in the following ratios:



$$\frac{AD}{\_\_\_} = \frac{\_\_\_}{AB}$$

8. Rewrite the proportion above as a new equation without fractions.

Have your teacher initial the box, before you continue.

9. **Measure** the following segments:

DB = _____ AC= _____
DA = _____ AE = _____

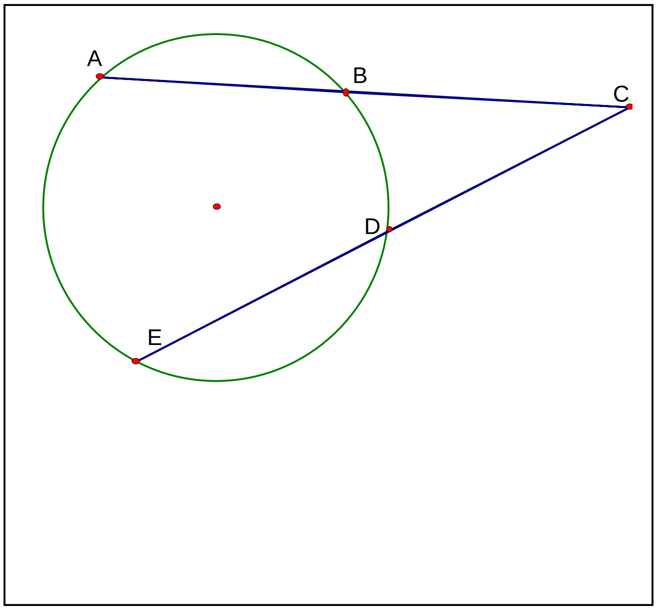
Using the Calculate option* in Sketchpad, verify your equation from step 9. Write your results here:

*In Sketchpad, Click on Measure and then on Calculate. A box that resembles a calculator will appear on your screen. Since you have already calculated the lengths of the segments, you can tell the calculator to perform functions on selected segments. Click on a segment's length and it will appear in the calculator.

10. Will this equation always hold true? Drag point A around the circle and observe your results. Explain.

11. In your own words, write a theorem describing when two secant segments are drawn to a circle from an external point.

12. Complete the following:



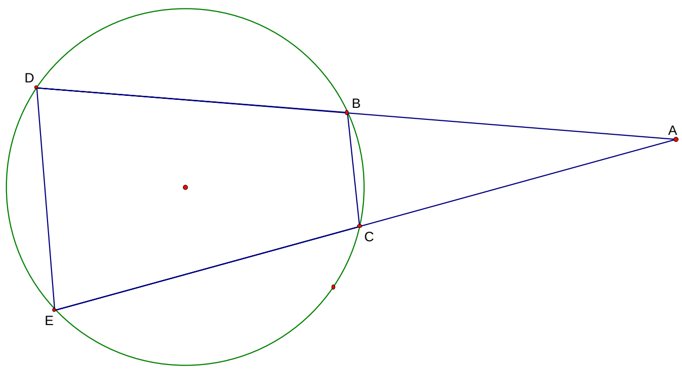
Using the diagram on the Left

1. If AC= 14, BC = 6, and EC= 21, then CD= ______.

2. IF CD= 5, DE= 7, BC= 4, then AC = __________.

Enrichment for Using Similar Triangles to Identify Secant Relationships Lesson.

To complete the following problem, you will need to use the diagram you constructed from worksheet #2.



1. What shape is formed within the circle?

2. Construct a perpendicular bisector to each of the four sides.
   What relationship does the center of the circle and this intersection have?

3. Start with a new sketch, construct two similar triangles with the following parameters:
   i. The smaller triangle is in the bigger triangle.
   ii. They share a common vertex.
   iii. Two sides can not be parallel.

4. Notice a quadrilateral is formed. Circumscribe a circle around the quadrilateral.

Print your results.

Tangents, Secants, and Chords…OH MY!

Development/Procedures:

Lesson #3: Using Similar Triangles to Identify Secant/Tangent Relationships

Overview: In this lesson, students will use Geometer's Sketchpad to demonstrate an application of similar triangles. Students discover the theorem that when a secant and tangent segment are drawn to a circle from an external point,
(secant segment * external segment) = $(\text{tangent segment})^2$. Students may work with a partner or individually.

(Preassessment/Launch 0-5 min) To begin the class, hand out Worksheet #3 to each student and discuss the review questions as a class.

(Teacher Facilitated/Student Application 30-40 min). Once the students have reviewed prerequisite concepts, have them continue the worksheet and work self-guided for the rest of the class period. The teacher should offer help if students have difficulty with the Sketchpad constructions, but should encourage them to work through the problems and make conjectures on their own.

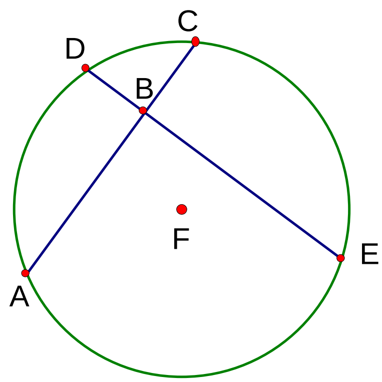
(Embedded Assessment) Monitoring student progress on the worksheet is an ongoing assessment. .

(Reteaching/Summarizing 0-5 min) To conclude the lesson, bring the class together and ask several students to share their measurements for their secant segments, tangent segments and the resulting products. Ask several students to verbalize their theorems and as a class, decide on a final statement of the theorem. Make sure that the final theorem is mathematically correct and clearly stated for all the students.

An Application is provided for students who finish early and/or for future class time.

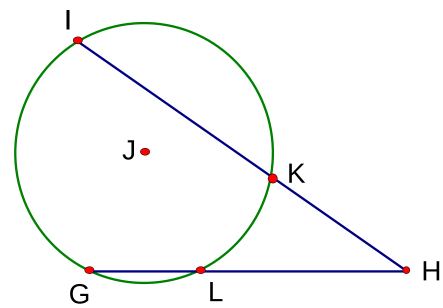
Worksheet #3: Using Similar Triangles to Identify Secant/Tangent Relationships

Review:



IF BD=2, DE= 12, and AC= 9,

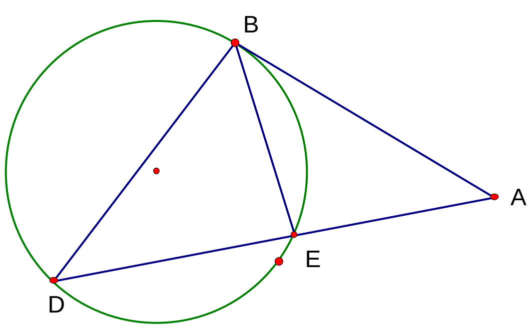
then AB= _____.



If KH= 5, KI= 7, and GL= 4,

then LH= _______.

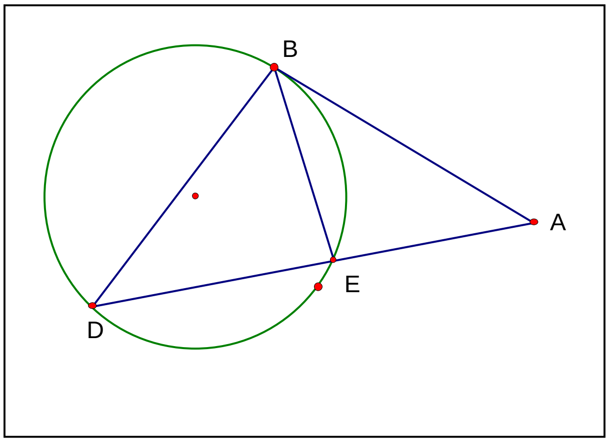
Complete the following sentences that pertain to the following diagram. BA is tangent to the circle at point B.



1. The measure of the inscribed angle <BDE= ½ ____________.
2. The measure of the angle <EBA= ½ __________________.

1. Open Geometer's Sketchpad.

2. Construct the following diagram.



Reminder: AB must be perpendicular to the radius whose endpoint is also B. If you have difficulty making this construction, draw a radius from the center point to B. Highlight the radius and point B, construct a perpendicular line. Then, construct the segment AB.

3. Your goal is to identify two similar triangles. According to the Angle-Angle Postulate, two corresponding angles need to be congruent. Which two sets of corresponding angles do you think are congruent?

<________ = < ____________ Why? ____

____

____

____

<________ = <____________ Why? ____

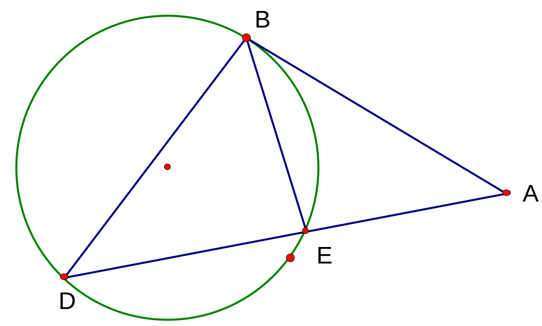
____

____

____

4. **Measure** the following angles:

<BAD = ________ <BAE = ________
<BEA = ________ <DBA= ________
<BDE= ________ <EBA = ________



Which pairs of angles are congruent?

Does this agree with your answer to question #3? If not make any necessary corrections to #3.

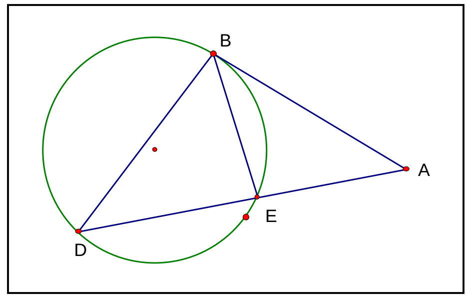
Is there a pairing that you did not identify in your answer to #3? Without using measurements, how can you justify their congruency?

5. Complete the following statement: $\triangle ABD \sim \triangle$______
Use mathematics to justify your statement.

6. From the definition of similar triangles, corresponding sides are in proportion.

Fill in the following ratios:

$$\frac{AD}{\quad} = \frac{\quad}{AE}$$



7. Rewrite the proportion above as a new equation without using fractions.

8. Measure the following segments:

AB = ______ AD = ______

AE = ______

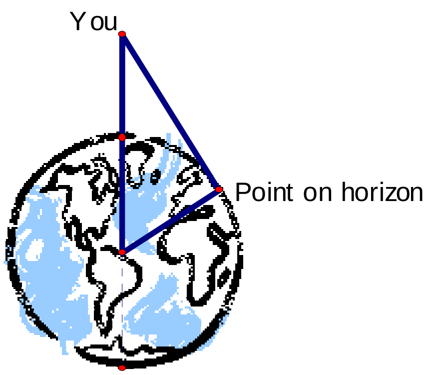
Using the Calculate option in sketchpad, verify your equation from step 9. Write your numerical results here:

9. Will this relationship always hold true? Drag point A around the circle and observe your results.

10. In your own words, write a theorem describing when a secant and tangent segment are drawn to a circle from an external point.

Application – Distance to the Horizon

You are a certain distance above the ocean looking out at the horizon. If you know the height of an object and the radius of the earth, you can actually calculate how far you can see. The following picture demonstrates this principle. Your line of sight is tangent to the radius of the earth.



Radius of Earth	~ 6378 km ~ 3963 miles
5280 feet = 1 mile	

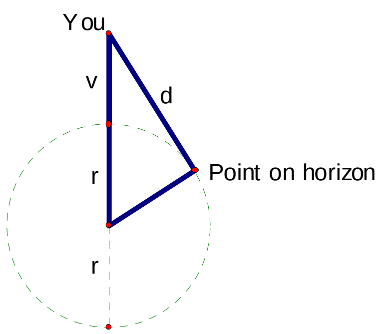
Using your tangent/secant segment relationships, solve the following problem.

1. You are in a hot air balloon and your eye level is 50 meters over the ocean. On a clear day, how far away is the farthest point you can see over the ocean?

_______ km

2. How could you have used the Pythagorean Theorem to solve the problem?

3. Apply the Pythagorean Theorem to the following picture to develop the tangent/secant relationship $d^2 = v\ (2r + v)$

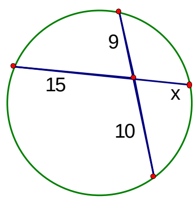


Name________________________________ Date________

Quiz – Tangents, Secants, and Chords……OH MY!

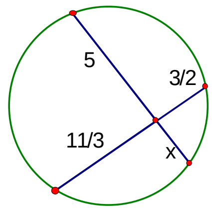
Solve for x

1.



x = ________

2.



x = ________

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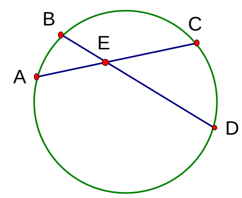
3. Use the following information to solve for the indicated segment:
Note: Figure not drawn to scale



AC = 20 cm
BE = 7 cm
ED = 12 cm

AE = ________

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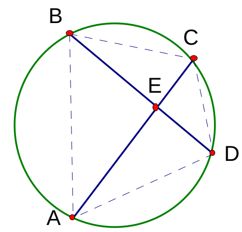
4. Which of the following triangles are similar? (Choose one)



a) △AEB and △AED

b) △AED and △CEB

c) △ABE and △DCE

d) △ABC and △ADC

e) △BEC and △DEC

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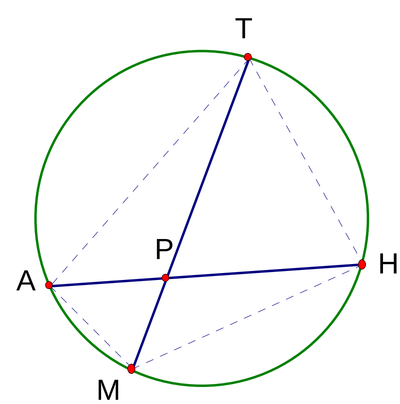
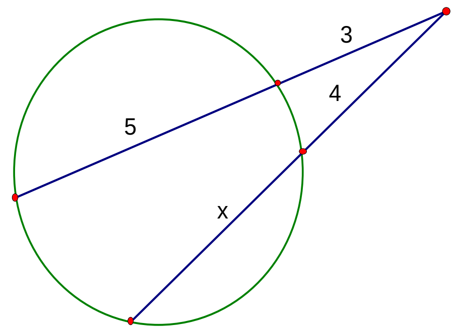
5. Identify two similar triangles and explain why they are similar.



6. Solve for x.
Note: Figure not drawn to scale



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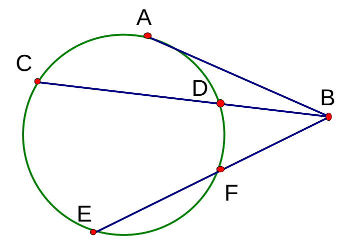
7. $\overline{AB}$ is tangent to the circle. Find the lengths indicated.



a) AB = 6; BD = 4; CD = ________

b) BF = 5; EF = 5; AB= _________

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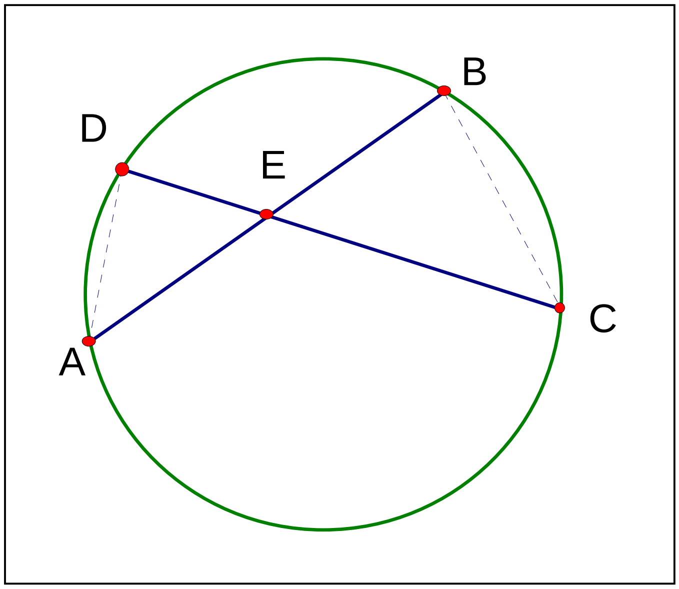
8. If you stand on a hill next to the ocean with your eyes 20 m above sea level, how far out over the ocean can you see? Round to the nearest hundredth. (Radius of the earth = 6378 km)

Worksheet #1: Using Similar Triangles to Identify Chord Relationships

Review (Complete the following sentences):

1. **The measure of internal angle <CEA = 1/2 (arc AC + arc DB).**
2. **Two inscribed angles that intercept the same arc are congruent.**

3. **If < ADC = 88^o, then arc AC = 176.**
4. **If arc DB= 114^o and <ADC= 88^o, then <DEB = 145.**



1. Open Geometer's Sketchpad.

2. Construct a circle.

3. Construct two intersecting chords that do NOT pass through the center of the circle.

4. Construct the point of intersection of the two chords. Label the endpoints and the point of intersection as shown in the diagram above.

5. Construct the segments that connect the endpoints of each chord as shown in the diagram. By doing this, you create two triangles.

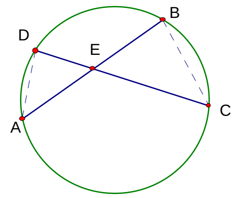
6. **Measure** the following angles and record your answers:
   <ADC = Answers will vary <DAB = Answers will vary
   <ABC = Answers will vary <DCB = Answers will vary

   Is there a relationship between the angles? How could you have predicted this result without measuring the angles?
   <ADC = <ABC <DAB = <DCB

<ADC and <ABC intercept the same arc, therefore they are congruent. This is also true for <DAB and <DCB.

What do you know about <AED and <CEB? Why?

They are congruent by vertical angle theorem



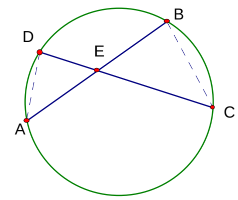
7. Complete the following statement: △DEA ~ △CEB
   Use mathematics to justify your statement.

Your students may want to explain in a two-column proof or paragraph form.

Statement	Reason
1. When two chords intersect in a circle	1. Given
2. <DEA is congruent to <BEC	2. Vertical Angle Theorem.
3. <ADC is congruent to <ABC	3. Two inscribed angles that intercept the same arc are congruent.
4. △AED ~ △CEB	4. AA Postulate

8. From the definition of similar triangles, corresponding sides are in proportion.

Fill in the following ratios:



$$\frac{CE}{AE} = \frac{BE}{DE}$$

9. Rewrite the proportion above as a new equation without fractions.

AE·BE=CE·DE

10. **Measure** the following segments:

CE = Answers will vary AE = Answers will vary
ED = Answers will vary EB = Answers will vary

11. Using the Calculate option* in Sketchpad, verify your equation from step 9. Write your results here:

Answers will vary

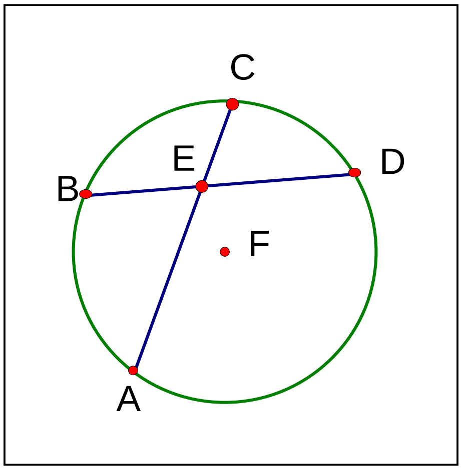
12. Will this relationship always hold true? Drag point A to a new point on the circle and observe your results. Explain.

Yes, because the triangles stay similar.

13. In your own words, write a theorem describing when two chords intersect in a circle.

When two chords intersect in a circle, the product of the segments of one chord equals the product of the segments of the other chord.

14. Try the following problems.

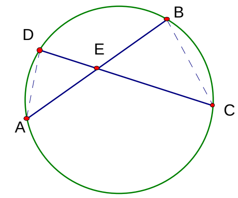


Using the diagram on the Left

1. If BE= 4, CE=3, and DE=9, then AE= _12__.

2. If CA= 14, BE= 4, and ED= 6, then AE=_2 or 12__.

*In Sketchpad, Click on Measure and then on Calculate. A box that resembles a calculator will appear on your screen. Since you have already calculated the lengths of the segments, you can tell the calculator to perform functions on selected segments. Click on a segment's length and it will appear in the calculator.



Extensions

1. Ratio of the Areas

a) How do you think the ratio of the areas of the two similar triangles compare to the scale factor?
Answers will vary

b) Using the same construction from Worksheet #1, calculate:

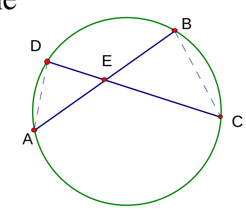
Area of △AED = _ Answers will vary ________

Area of △CEB = _ Answers will vary ________

Ratio of Areas = $\frac{\text{Area of } \triangle AED}{\text{Area of } \triangle CEB}$ = Answers will vary

Scale Factor = Answers will vary

c) Do you notice a relationship between the ratio of the areas of the triangles and the scale factor? If you do not see one, try experimenting with powers and exponents.
They should see that the ratio of the areas is the scale factor squared.



2. Maximum and minimum chord products

a) Using your construction, create a table that includes the following segment measurements: AE, EB, DE, EC, AE*EB and DE*EC. Create a total of three different rows by moving the chords around. Observe the change in product values.

AE	**EB**	**DE**	**EC**	**AE·EB**	**DE·EC**

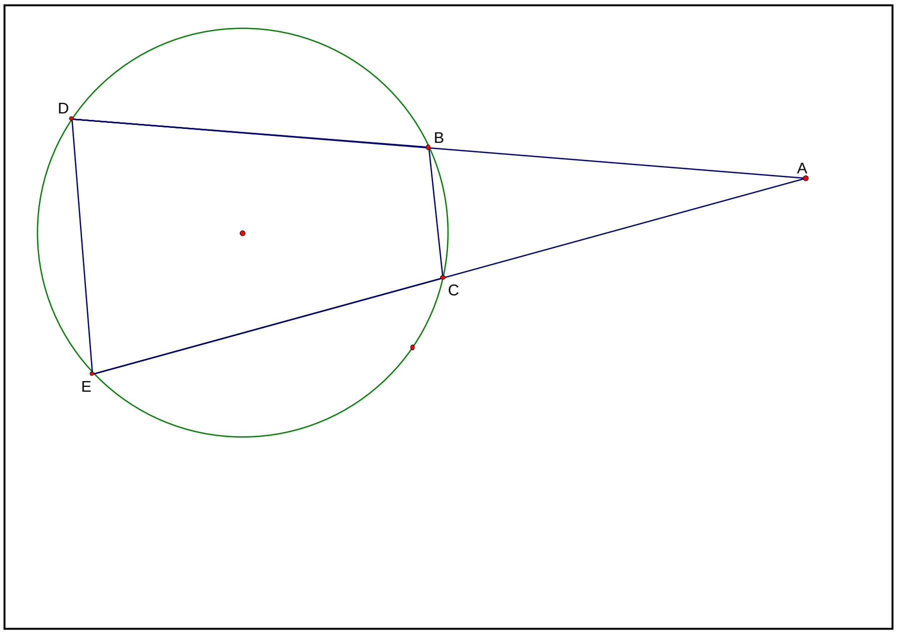
Answers will vary

b) Continue to move the endpoints of the chords around the circle until you obtain the maximum value for **AE·EB** and **DE·EC**. Add this row to your table above. What do you notice about AE, EB, DE, and EC?
That AE,EC, DE, and EC are radii. They should notice that the intersection becomes the center of the circle.

c)Complete the following sentences. When the product of the chords is at the greatest value, the chord segments are also radii. Point __E__ is the center of the circle.

Worksheet #2: Using Similar Triangles to Identify Secant Relationships

Review: The measure of inscribed angle <EDB = 1/2 __arc ECB_.
The measure of angle <DEC = ½ (_arc DBC__).
A circle has __360_______ degrees.
A semicircle has _180_________degrees.
The sum of the angles in a triangle is ____180_____ degrees.
A straight angle is ____180_______ degrees.
Two angles whose measures sum to 180 are _supplementary_.



1. Open Geometer's Sketchpad.

2. Construct the diagram above.

3. **Complete** the following:

a. <EDB intercepts arc__ECB__.
b. <ECB intercepts arc __EDB___.

The sum of the two arcs is 360_ degrees.

c. < DEC intercepts arc __DBC___.
d. <DBC intercepts arc __DEC__.

The sum of the two arcs is __360__ degrees.

4. **Measure** the following angles:

<EDB = Answers will vary
<ECB = Answers will vary
<DBC = Answers will vary
<DEC = Answers will vary

What relationship did you discover? How could you have predicted this without measuring the angles?

<EDB and <ECB are supplementary as well as <DBC and DEC. The answers may vary. The arcs equal 360, when divided by 2 equals 180.

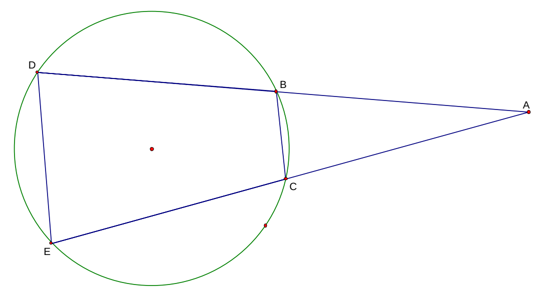
5. **Measure** the following angles:

<EDA= Answers will vary <DEA= Answers will vary

<BCA= Answers will vary <CBA Answers will vary

What relationship did you discover? How could you have predicted this without measuring the angles?

<EDA and <BCA are congruent as well as <DEA and <CDA. This is true because when two angles are supplements of congruent angles, then the two angles are congruent.



6. Complete the following statement:

△ADE ~ △ACB.

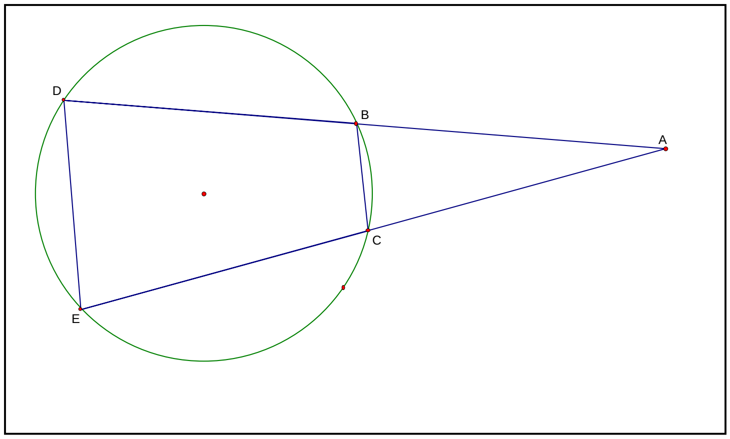
Use mathematics to justify your statement.

Your students may want to explain in a two-column proof or paragraph form.

Statement	Reason
1. AD and AE are secant segments.	1. Given
2. <DAE ≅ <BAC	2. Reflexive
3. <EDB ≅ ½ arc ECB	3. An inscribed angle is half its intercepted arc.
4. <ECB ≅ ½ arc EDB	4. An inscribed angle is half its intercepted arc.
5. arc ECB + arc EDB= 360^{o}	5. Arc Addition Postulate
6. ½ (arc ECB + arc EDB)= 180^{o}	6. Division
7. ½ arc ECB + ½ arc EDB = 180^{o}	7. Distribution
8. <EDB + <ECB = 180^{o}	8. Substitution
9. <ECB + <BCA = 180^{o}	9. Angle Addition Postulate
10. <ECB = 180^{o} - <BCA	10. Subtraction
11. <EDB + (180^{o} - <BCA) = 180^{o}	11. Substitution
12. <EDB ≅ <BCA	12. Subtraction
13. △DAE ~ △CAB	13. AA Postulate

7. From the definition of similar triangles, corresponding sides are in proportion.

Fill in the following ratios:



$$\frac{AD}{AC} = \frac{AE}{AB}$$

8. Rewrite the proportion above as a new equation without fractions.

AD·AB=AE·AC

Have your teacher initial the box, before you continue.

9. **Measure** the following segments:

DB = Answers will vary AC= Answers will vary
DA = Answers will vary AE = Answers will vary

Using the Calculate option* in Sketchpad, verify your equation from step 9. Write your results here:
Answers will vary

*In Sketchpad, Click on Measure and then on Calculate. A box that resembles a calculator will appear on your screen. Since you have already calculated the lengths of the segments, you can tell the calculator to perform functions on selected segments. Click on a segment's length and it will appear in the calculator.

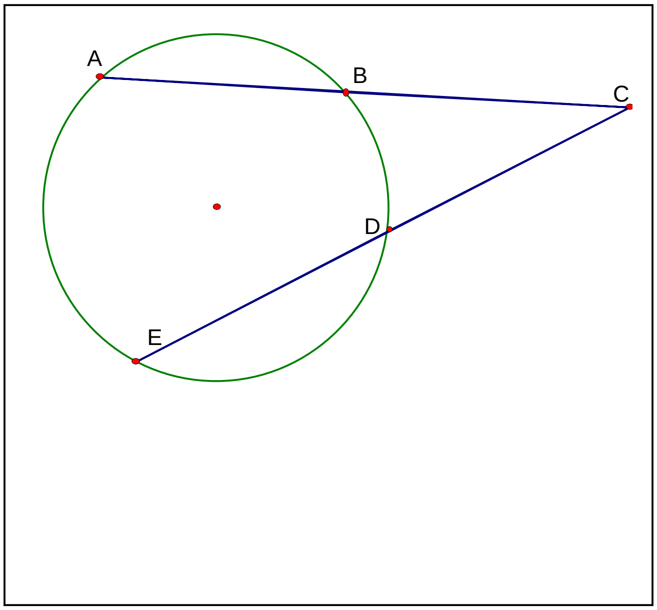
10. Will this equation always hold true? Drag point A around the circle and observe your results. Explain.

<u>The similar triangles stay proportional.</u>

11. In your own words, write a theorem describing when two secant segments are drawn to a circle from an external point.

<u>When two secant segments are drawn to a circle from an external point, the produce of one secant segment and its external segment equals the product of the other secant segment and its external segment</u>

12. Complete the following:



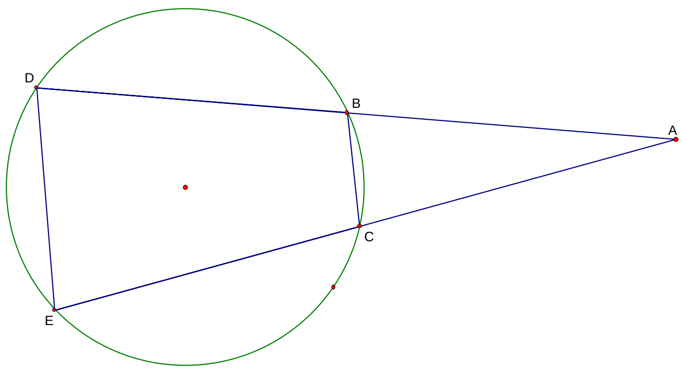
Using the diagram on the Left

1. If AC= 14, BC = 6, and EC= 21, then CD= __4__.

2. IF CD= 5, DE= 7, BC= 4, then AC = __15_____.

Enrichment for Using Similar Triangles to Identify Secant Relationships Lesson.

To complete the following problem, you will need to use the diagram you constructed from worksheet #2.



1. What shape is formed within the circle?

   Quadrilateral

2. Construct a perpendicular bisector to each of the four sides.
   What relationship does the center of the circle and this intersection have?

   The center of the circle and the intersection are the same point.

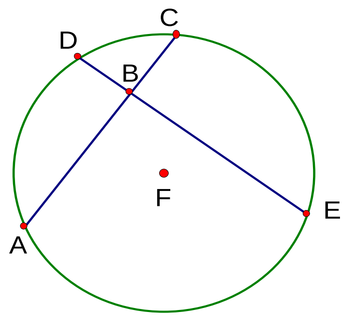
3. Start with a new sketch, construct two similar triangles with the following parameters:
   i. The smaller triangle is in the bigger triangle.
   ii. They share a common vertex.
   iii. Two sides can not be parallel.

4. Notice a quadrilateral is formed. Circumscribe a circle around the quadrilateral.

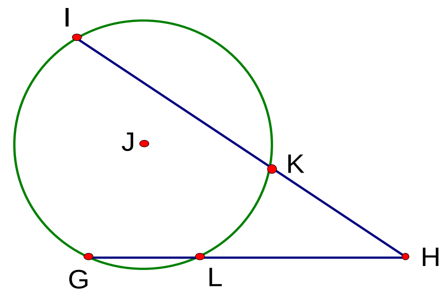
Print your results.

Worksheet #3: Using Similar Triangles to Identify Secant/Tangent Relationships

Review:

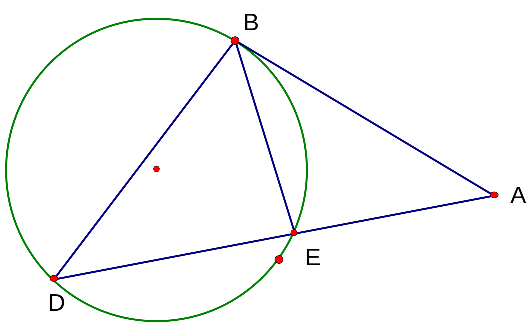


If BD = 2, DE = 12, and AC = 9
then AB = 4 or 5



If KH = 5, KI = 7 and GL = 4
then LH = _6_

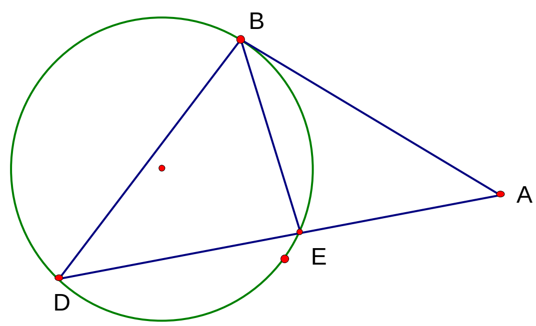
Complete the following sentences that pertain to the following diagram. BA is tangent to the circle at point B.



1. The measure of the inscribed angle <BDE= ½ Arc EB.
2. The measure of the angle <EBA= ½ Arc EB.

1. Open Geometer's Sketchpad.

2. Construct the following diagram.



Reminder: AB must be perpendicular to the radius whose endpoint is also B. If you have difficulty making this construction, draw a radius from the center point to B. Highlight the radius and point B, construct a perpendicular line. Then, construct the segment AB.

3. Your goal is to identify two similar triangles. According to the Angle-Angle Postulate, two corresponding angles need to be congruent. Which two sets of corresponding angles do you think are congruent?

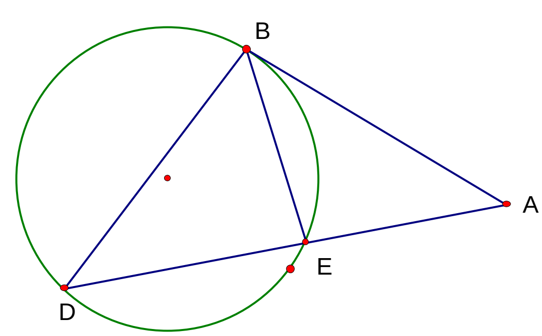
Answers may vary. Here is a sample answer:

a) <BDE = <EBA Why? An angle created by a tangent and a chord is ½ its intercepted arc. Thus <EBA = ½ Arc EB. An inscribe angle = ½ its intercepted arc so < BDE = ½ Arc EB. Thus the two angles are congruent.

b) <BAE = <BAD Why? reflexive

4. **Measure** the following angles: Answers may vary

<BAD = ________	<BAE = ________
<BEA = ________	<DBA= ________
<BDE= ________	<EBA = ________



Which pairs of angles are congruent?

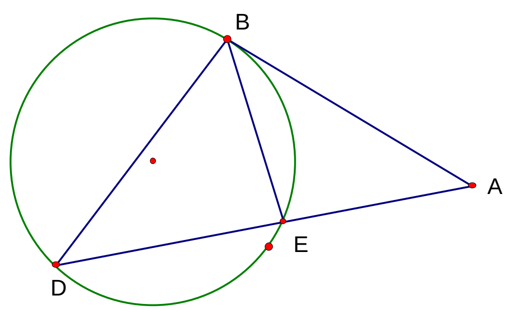
< BAD = < BAE
< BEA = < DBA
< EBA = < BDE

Does this agree with your answer to question #3? If not make any necessary corrections to #3. Answers may vary

Is there a pairing that you did not identify in your answer to #3? Without using measurements, how can you justify their congruency?
Answers may vary. Sample answer:

Yes, I did not identify < BEA and < DBA.
<BEA and <DBA are located in two triangles that have two corresponding congruent angles. Thus <BEA and <DBA are also congruent.

5. Complete the following statement: △ABD ~ △AEB
Use mathematics to justify your statement.



Your students may want to explain in a two-column proof or paragraph form.

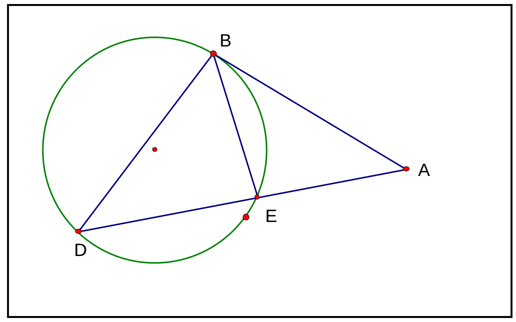
Given: AB is tangent to the circle
AE is a secant segment

Prove: △ABD ~ △AEB

Statements	Reasons
1. AB is tangent to the circle AE is a secant segment	1. Given
2. <BDE = ½ Arc BE	2. An inscribed angle = ½ its intercepted arc.
3. <ABE = ½ Arc BE	3. The angle formed by a tangent and a chord = ½ its intercepted arc.
4. <ABE ≅ <BDE	4. Substitution
5. <BAE ≅ < BAD	5. Reflexive
6. △ABD ~ △AEB	6. Angle-Angle Similarity

6. From the definition of similar triangles, corresponding sides are in proportion.

$$\frac{AE}{AB} = \frac{AB}{AD}$$



7. Rewrite the proportion above as a new equation without using fractions.
AB * AB = AD * AE

AB^2 = AD * AE

8. Measure the following segments: Answers may vary

AB = _____ AD = _____
AE = _____

Using the Calculate option in sketchpad, verify your equation from step 9. Write your numerical results here:

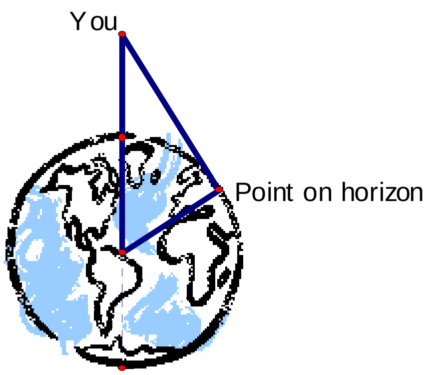
9. Will this relationship always hold true? Drag point A around the circle and observe your results.
Yes, the relationship always holds true.

10. In your own words, write a theorem describing when a secant and tangent segment are drawn to a circle from an external point.

When a secant and a tangent segment are drawn to a circle from an external point, the product of the external segment of the secant and the entire secant segment equals the square of the tangent segment.

Application – Distance to the Horizon

You are a certain distance above the ocean looking out at the horizon. If you know the height of an object and the radius of the earth, you can actually calculate how far you can see. The following picture demonstrates this principle. Your line of sight is tangent to the radius of the earth.



Radius of Earth	~ 6378 km ~ 3963 miles
5280 feet = 1 mile	

Using your tangent/secant segment relationships, solve the following problem.

1. You are in a hot air balloon and your eye level is 50 meters over the ocean. On a clear day, how far away is the farthest point you can see over the ocean?

$(\text{distance})^2$ = external segment * entire secant segment

$d^2 = .05 \text{ km} * (12756.05\text{km})$

$d^2 = 637.8 \text{ km}^2$

<u>25.25</u> km $\quad d = 25.25$ km

2. How could you have used the Pythagorean Theorem to solve the problem? Verify your solution to #1.

The tangent segment creates a right angle with the radius.
If d = distance to horizon, r = radius, and v = vertical height of object:

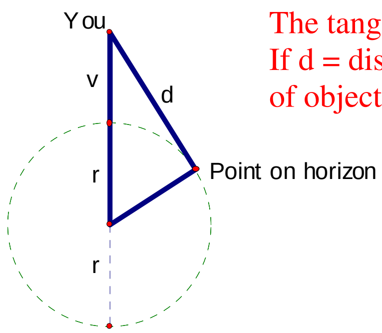
$d^2 + r^2 = (r + v)^2$

$d^2 + 40678884 \text{ km}^2 = 40679521.80 \text{ km}^2$

$d^2 = 637.8 \text{ km}^2$

$d = 25.25$ km

3. Apply the Pythagorean Theorem to the following picture to develop the tangent/secant relationship $d^2 = v\,(2r + v)$



The tangent segment creates a right angle with the radius.
If d = distance to horizon, r = radius, and v = vertical height of object:

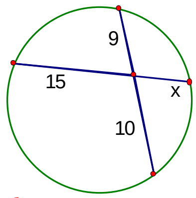
$$d^2 + r^2 = (r + v)^2$$
$$d^2 + r^2 = r^2 + 2rv + v^2$$
$$d^2 = 2rv + v^2$$
$$d^2 = v\,(2r + v)$$

Name__**Teacher's Key**____________ Date________

Quiz – Tangents, Secants, and Chords……OH MY!

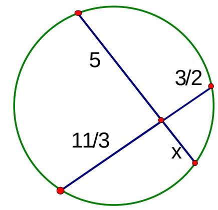
Solve for x

1.



x = __**6**______

2.



x = ___**11/10**_____

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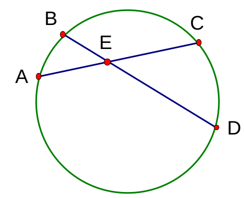
3. Use the following information to solve for the indicated segment:
Note: Figure not drawn to scale



AC = 20 cm
BE = 7 cm
ED = 12 cm

AE = _**6 or 14**__

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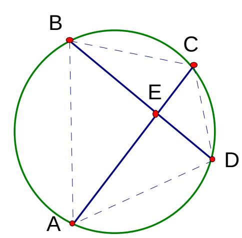
4. Which of the following triangles are similar? (Choose one)



a) △AEB and △AED

b) △AED and △CEB

c) △ABE and △DCE **(C)**

d) △ABC and △ADC

e) △BEC and △DEC

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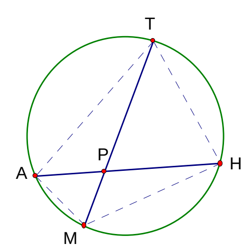
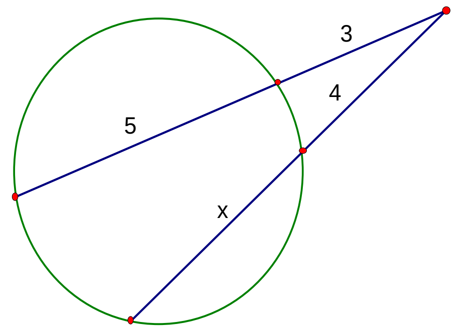
5. Identify two similar triangles and explain why they are similar.



ΔAED~ΔBEC because of the AA postulate. <CAD is congruent to <CBD because they intercept the same arc.<CEB is congruent to <AED because of the vertical angle theorem. The students' answers may vary.

6. Solve for x
Note: Figure not drawn to scale
X= 2____



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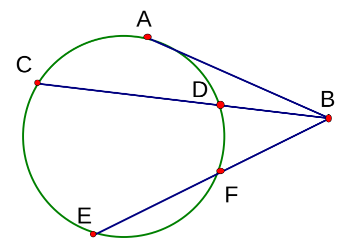
7. $\overline{AB}$ is tangent to the circle. Find the lengths indicated.

a) AB = 6; BD = 4; CD = __**5**___

b) BF = 5; EF = 5; AB= __**5√2**__



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8. If you stand on a hill next to the ocean with your eyes about 20 m above sea level, how far out over the ocean can you see? Round to the nearest hundredth. (Radius of the earth = 6378 km)

d^2 = (external segment) * (entire secant segment)
d^2 = (.02 km)* (12756.02 km)
d^2 = 255.12 km^2
d = 15.97 km